Chapter 2: Signal Analysis

Chapter 2 Objectives

At the conclusion of this chapter, the reader will be able to…

- Explain the difference between the frequency and time domains.
- Draw a diagram of a sine wave in both the frequency and time domains.
- Explain how a complex waveform (such as a square wave) looks in the frequency domain.
- Define the terms fundamental and harmonic.
- Define noise, and list at least two internal and two external noise sources.
- Calculate the signal-to-noise ratio if the signal and noise voltages (or powers) are known.
- Explain how the noise figure is calculated for an amplifier.

Many different kinds of signals are produced and used by electronic systems. A *signal* is an electrical current or voltage that either represents information (the *information signal*) or performs some useful function (the *carrier signal* in radio). Most signals are alternating-current sine waves, or as we shall see, combinations of sine waves. Signals can be viewed in either the *time* or *frequency* domains.

It's important for technicians to be able to accurately measure electrical signals. We can't see electrons flowing in circuits. Test equipment is our eyes. We will make almost all of our decisions based on what we read from test equipment -- so we must read it accurately!

Many systems have signals that are not wanted. The name for any unwanted signal is *noise*. Noise is produced both inside and outside of circuits. Radio receivers are especially sensitive to noise, as they must amplify extremely tiny signals from receiving antennas.

2-1 Two Domains

Time and Frequency Domains

There are two domains in which we can view electronic signals. They are the *time domain* and the *frequency domain*. All signals have a time and frequency domain representation. To see a signal in the time domain, we use an oscilloscope; to see it in the frequency domain, we use a different instrument called a *spectrum analyzer*. The pictures we get from each domain are quite different, but both are very important for understanding how communication circuits work.

Most technicians are very familiar with the instrument of Figure 2-1. This is, of course, an oscilloscope. An oscilloscope has a trace that sweeps across the screen from left to right at a selected and calibrated rate. As the voltage of a waveform varies from positive to negative and back over time, a waveform results.

Oscilloscopes display signals in the Time Domain.

A scope works a lot like the popular "Etch-A-Sketch" toy. The *timebase* of a scope moves the dot (horizontal knob on the toy) across the face of the screen at a constant speed. The *vertical deflection* section of the scope moves the dot up and down (same as the vertical knob). Because a scope does this over and over at a rapid rate, our eyes fuse the images together into one continuous line.

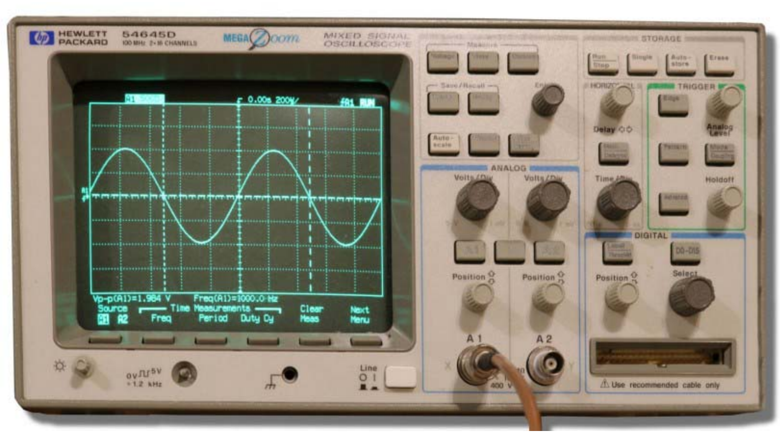

Figure 2-1: A Typical Oscilloscope

We could say that a scope *draws a waveform as it happens*. In other words, an oscilloscope shows pictures of waveforms in the *time domain*. The horizontal axis on an oscilloscope is in units of *time*, in *seconds*.

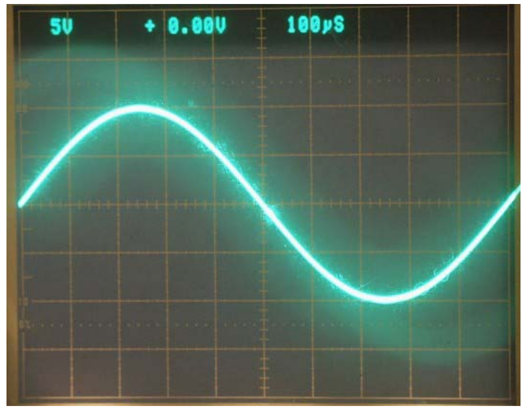

Figure 2-2: Reading the Oscilloscope

Example 2-1

What is the *frequency*, *peak voltage, peak-to-peak voltage*, and *RMS (root mean square or effective) voltage* of the waveform pictured in Figure 2-2, if the scope settings are as follows:

Horizontal, 100 σs per division.
Vertical, 5 V per division.

If this voltage is being measured across a 50 T resistor, what power will result?

Solution:
The frequency can be calculated if the *period* (T) is known:

$$f \mid \frac{1}{T} \mid \frac{1}{1ms} \mid \underline{\underline{1000 Hz}} \mid \underline{\underline{1\,kHz}}$$

The peak voltage can be calculated by observing that the trace goes two grid squares above (or below) the baseline at the peak of each cycle:

$$Vpk \mid (2\ divisions)(5V / division) \mid \underline{\underline{10\ Vpk}}$$

The peak-to-peak voltage is the total height of the waveform:

$$Vpp \mid (4\ divisions)(5V / division) \mid \underline{\underline{20\ Vpp}}$$

The *RMS* or *effective* value of the waveform can be calculated since the shape is a sine wave:

$$Vrms \mid \frac{Vpk}{\sqrt{2}} - 0.707Vp - (0.707)(10V) - \underline{\underline{7.07\ V}}$$

Note that many people even use 0.7 (rather than 0.707) as an "approximate" factor for calculating an RMS voltage. When reading from an oscilloscope, this is quite valid, since there may be as much as 5% measurement error just from "eyeballing" the display!

Caution: The formula above for RMS voltage is only valid for a sine wave!

The *power* can be calculated using Ohm's law:

$$P \mid \frac{V^2}{R} \mid \frac{7.07V^2}{50\ \mathrm{T}} \mid \underline{\underline{1\,W}}$$

Caution: To calculate power, an RMS voltage must be used!

A Shortcut for Calculating Power

If you know the peak-to-peak reading of a *sine wave* waveform, you can also calculate power by using:

$$P \mid \frac{Vpp^2}{8R} \mid \frac{20Vpp^2}{(8)(50\ \mathrm{T})} \mid \underline{\underline{1\ watt}}$$

Some people like this formula better, since it avoids the need to convert to RMS first. However it can only be used for sine waves!

The Frequency Domain

There's another way of looking at electrical signals. An oscilloscope shows signals in the *time domain*, which is fine for many types of measurements. However, many times we're much more interested in what *frequency* or *frequencies* a waveform might contain. An instrument that shows the *frequency domain content* of a signal is called a *spectrum analyzer.*

Being able to measure the *frequency content* of signals is very useful to us for several reasons. You'll recall that one of a radio receiver's tasks is to separate the one desired carrier signal from all the others on the air. This is only possible because each radio transmitter uses a different frequency. *The frequency content of a signal therefore determines whether or not it will be reproduced in a receiver!*

Second, there is only a finite amount of space on the bands in which to operate radio transmitters. Transmitters use up this space in the same way that the parking lot at the local mall fills up with cars. Two radio stations can't share the same frequency, or they'll interfere with each other. *By looking at a radio transmitter's signals in the frequency domain, we can determine its bandwidth. Bandwidth* is the amount of "frequency space" taken up by a signal. It's very much like the width of a motorcycle, car, or truck to be parked in a stall.

Last, we can often tell much more about the quality of a signal by looking at it in the frequency domain. Defects or *distortions* of a waveform are often hard to spot on a scope, especially if the waveform is a sine wave. As you'll see, any distortion in a sine wave will cause new frequencies called *harmonics* to appear. *Distortion of a sine wave (such as an RF carrier) is often much easier to spot on a spectrum analyzer than a scope.*

The Frequency Domain is Nothing New!

Long before the dawn of electronics, humans employed the concept of the frequency domain as shown in Figure 2-3.

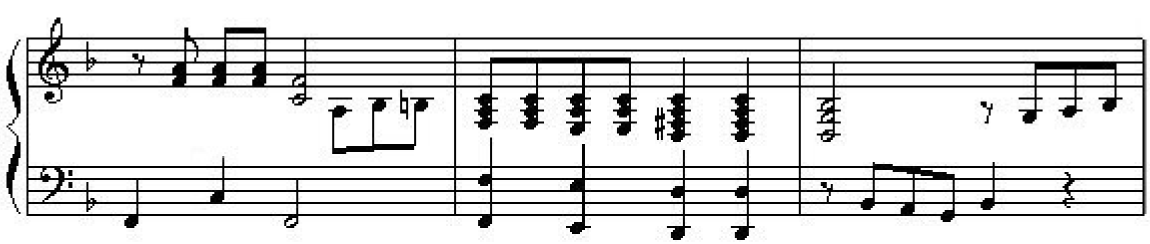

Figure 2-3: Old Time Frequency Domain Notation

Music notation refers to both *pitch* and *duration* for each note to be played. *Pitch* is really the listener's mental perception of the *frequency* of the note being played. Raise the frequency, and the listener hears a higher pitch. The vertical position of each note gives a precise indication of its pitch. Pitch is related to frequency. Therefore, we're looking at a *frequency-domain* picture here! (There is also time-domain information, because each different type of note plays for a unique time interval.)

Viewing Frequency Domain Information

A *spectrogram* is a graph showing the frequency-domain information of a signal. If the 1 kHz signal of Example 2-1 is a perfect sine wave, it will look like Figure 2-4 on a spectrogram.

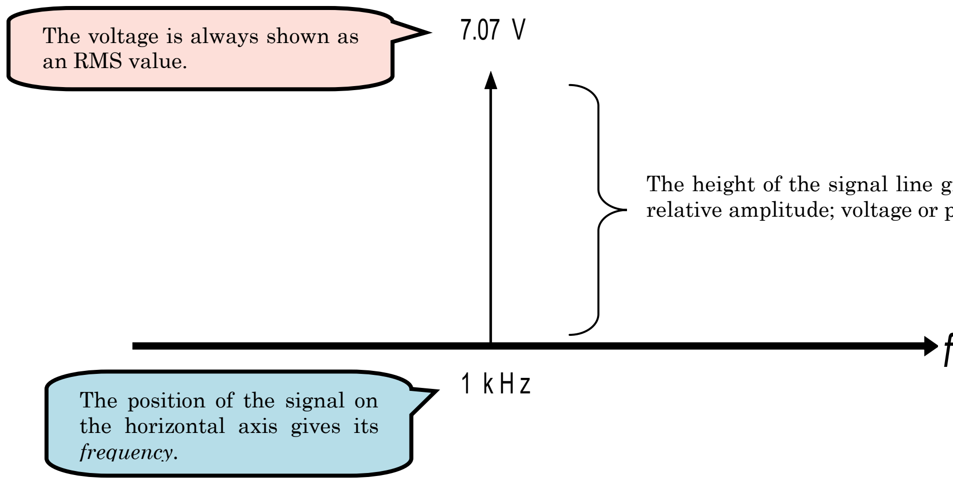

Figure 2-4: The Spectrogram of an Ideal Sine Wave

This picture is troubling; it doesn't look *anything* like a sine wave! It's just like the music above; the notation on paper doesn't *look or sound* anything like the music when it's being played, yet it still represents it. Figure 2-5 is the sine wave of Figure 2-1 displayed on a spectrum analyzer display.

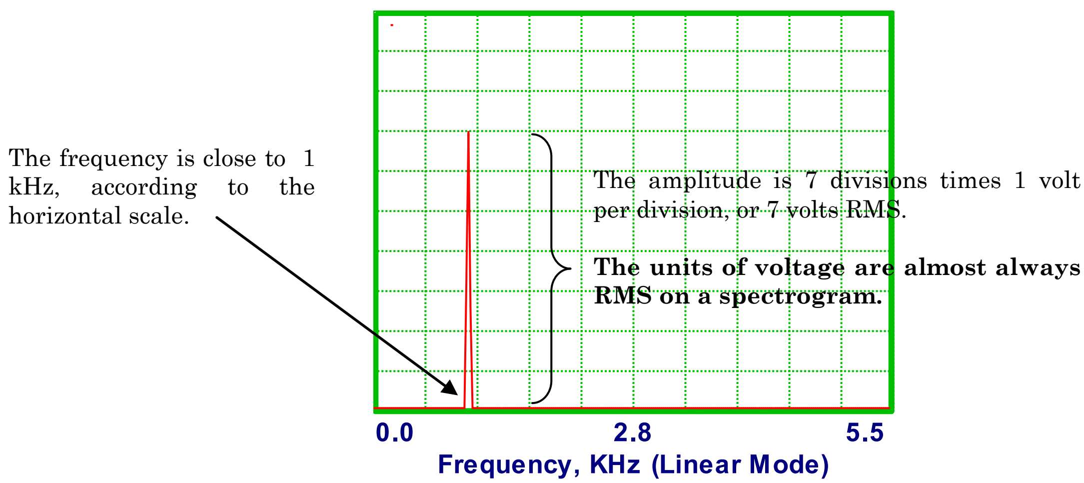

Figure 2-5: Spectrum Analyzer Display of the Pure Sine Wave
(Vertical setting, 1 V/division; Horizontal, 550 Hz/Division)

The sine wave is sometimes referred to as the only "pure" waveform, because a sine wave has only <u>one</u> frequency when it is viewed in the frequency domain.

Example 2-2

What type of waveform is being displayed below in Figure 2-6? Determine its frequency, RMS voltage, and peak voltage.

Solution:
The waveform displayed is another sine wave. A pure sine wave always shows up as one "line" on a spectrum analyzer display. By reading its position on the horizontal axis, we can see that the frequency is 2.8 kHz. The line is 4 units high, so its voltage is:

$$V \mid (4\,divisions)(1\,V/division) \mid \underline{\underline{4\,V}}$$

This is an RMS voltage. Therefore:

$$Vpk \mid Vrms\sqrt{2} - \frac{Vrms}{0.707} - \underline{\underline{5.66\,Vpk}}$$

Note that dividing by 0.707 is the same thing as multiplying by 1.41, which is approximately the square root of 2.

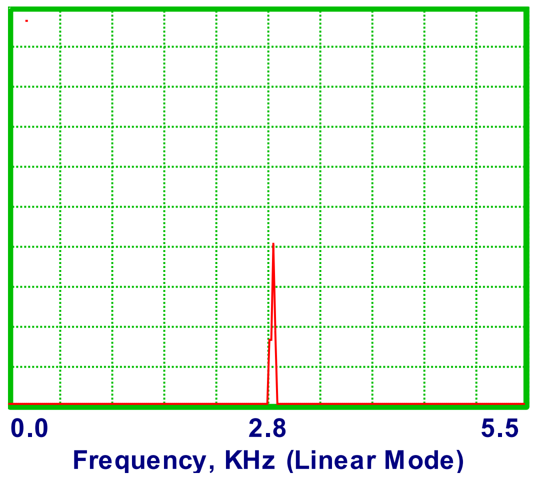

Figure 2-6: A Spectrogram Display
(Vertical setting, 1 V/division; Horizontal, 550 Hz/Division)

Section Checkpoint

2-1 What is a *signal?*
2-2 What are the two *domains* for viewing signals?
2-3 What instrument displays signals in the time domain? What are the units of its horizontal axis?
2-4 What is a spectrogram?
2-5 Give three reasons why technicians need to understand the frequency domain.
2-6 What instrument shows frequency domain information? What are the units on its horizontal axis?
2-7 What does a pure sine wave look like on a spectrogram?
2-8 Why is a sine wave referred to as "the only pure waveform?"
2-9 If the frequency of a sine wave is increased, what happens to its spectrogram picture?
2-10 If the amplitude (or voltage) of a sine wave is increased, how will its spectrogram change?

2-2 Complex Waveforms

Not all waveforms in electrical circuits are pure sine waves. A *complex waveform* is any signal that is not a sine wave. Don't be intimidated by the phrase "complex waveform." They're really not complicated at all! In this application, the word *complex* could be interpreted as *"made from many parts."* It's very likely that you've measured one or more of the following signals in the electronics lab:

∉ A square, triangle, or sawtooth wave

These are all complex and *periodic (repeating)* signals. *We say they're complex because they contain more than a single pure sine wave.* In communications, you're also likely to measure some of these as well:

∉ Human voice
∉ Music
∉ Digital data

Yes -- these signals are also complex waveforms. In fact, they're quite complicated, especially the first two. Speech and music signals are very hard to predict because they are not periodic (repeating). Fortunately, there's no need for an in-depth analysis of these signals to understand how they'll be carried by a communication system. We'll examine a few samples of these signals near the end of this section.

Fourier Analysis

A 19th century mathematician, Jean Baptiste Fourier (pronounced **four-ee'-ay**), was very interested in describing the movement of heat by using mathematics. The equations Fourier developed were *periodic* or *repeating*, but were not shaped like sine waves.

Fourier hypothesized that any periodic mathematical function (we can say electrical signal) could be represented by the addition of an *infinite series* (sum of an infinite number of terms) of *sine* and *cosine* (sine with 90-degree angle) waves, plus a DC level or average. This idea is very important in communications.

Any periodic signal that is *not a pure sine wave* can be considered to be built out of the following components:

- A DC component or "average" (which can be zero.)
- A *fundamental* sine wave that has a frequency *exactly* the same as the frequency of the signal.
- An *infinite* number of *harmonics*. A *harmonic* is a frequency that is an exact multiple of the fundamental frequency.

This is pretty heavy theory, so let's put it to work. We've seen what a sine wave looks like in both the time and frequency domains. By doing a "Fourier Analysis" of any waveform, we get its *frequency domain* picture. Figure 2-7 shows a 1 kHz, 1 V peak square wave.

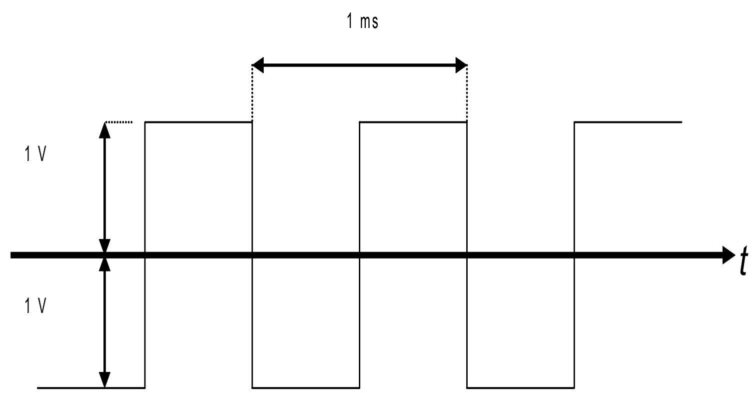

Figure 2-7: Time Domain (Oscilloscope) Picture of a 1 kHz Square Wave

If we were to do a Fourier analysis of this signal, we would get the *frequency domain* version. It would look something like Figure 2-8.

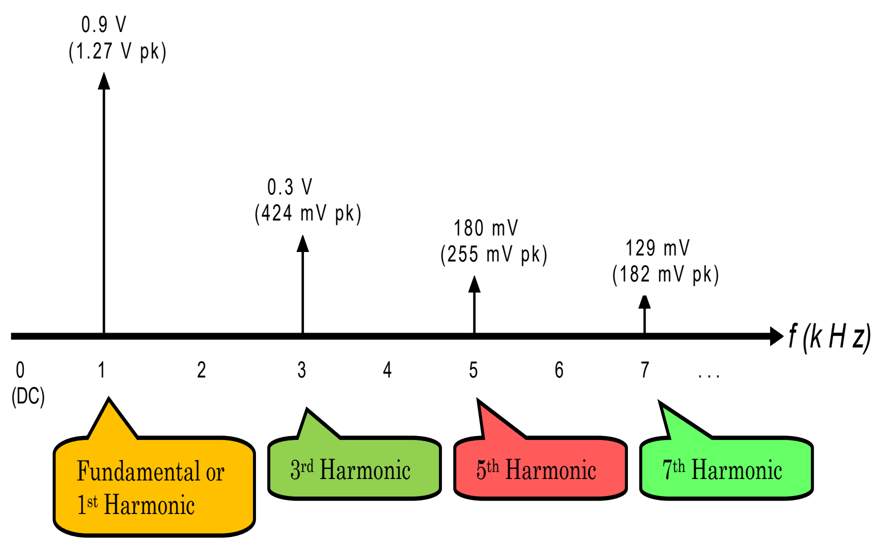

Figure 2-8: Frequency Domain Picture of a Square Wave

Again, this picture doesn't look *anything* like a square wave--but that's what it represents! By looking at this picture, we can tell that the 1 kHz square wave is really made of the following frequency components:

- A 1 kHz, 0.9 V (RMS) sine wave. (The *fundamental* or *first harmonic*)
- A 3 kHz, 0.3 V sine wave. (The *third harmonic*)
- A 5 kHz, 0.18 V sine wave (The *fifth harmonic*)
- A 7 kHz, 0.129 V sine wave (The *seventh harmonic*)

If you're curious about how the values of Figure 2-8 are actually calculated, Section 2-6 shows how that is accomplished.

This sequence continues to infinity. *In a perfect square wave, there are an infinite number of frequencies present.* Fortunately, the square waves in practical circuits aren't perfect, so we don't need to analyze an infinite number of frequencies to understand them. *In fact, we only need to analyze up to the 13th harmonic in order to get a decent reproduction of a square wave.*

Notice that the voltages of the sine wave frequencies gradually get smaller as frequency is increased. Eventually, they approach zero. This is why most techs use the rule about the 13th harmonic when dealing with square waves. Don't be concerned with *how* we obtained the voltages; just keep in mind that they will tend to get smaller as frequency increases.

Something is Missing!

Missing Harmonics in Signals and Waveform Symmetry

You'll notice several things missing from Figure 2-8, namely the harmonics at 2 kHz, 4 kHz, 6 kHz (and so on) and the DC Level. Where are they?

The *even harmonics* are not present because the square wave is perfectly *symmetrical*. The bottom and top look just like each other; they're just *mirror images*. *Any waveform with this type of symmetry will have only odd-numbered harmonics*. We say that the even harmonics have been cancelled out.

The *DC level* is absent because the average of the signal voltages in Figure 2-7 is *zero*. The signal spends exactly the same amount of time being positive as it spends being negative, so on average, the DC voltage will be zero. If this were not true, the signal would look like Figure 2-9 below:

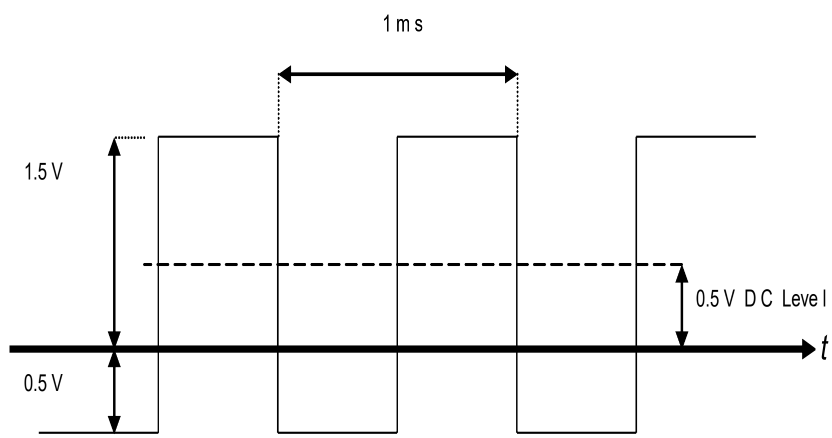

Figure 2-9: A Square Wave Riding on a DC Level

The signal of Figure 2-9 is still a 2 V peak-to-peak square wave (1 V peak), however, it been pushed up or "clamped" to a level of 0.5 V DC. The frequency-domain version of Figure 2-9 is shown in Figure 2-10.

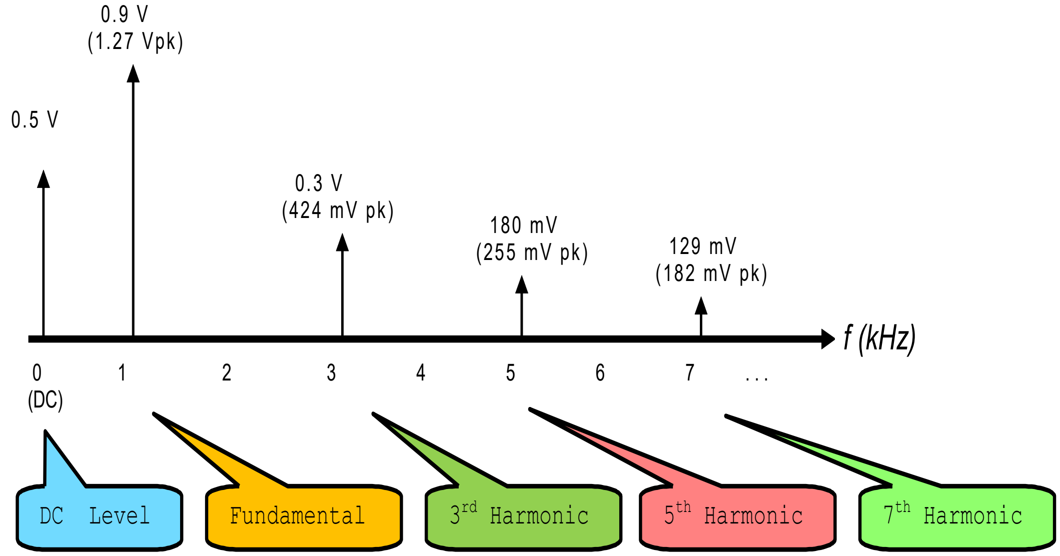

Figure 2-10: Frequency Domain Picture of the Square Wave Riding on a DC Level

As you can see, the only difference between Figure 2-8 and Figure 2-10 is the addition of the 0.5 Volt DC level. *Adding or subtracting a DC level from a complex waveform doesn't affect the sine wave frequency amplitudes.*

Square Wave Signal Measurements

Figures 2-11 and 2-12 show an oscilloscope and spectrum analyzer view of the same 1 kHz square wave that we just analyzed. How do the pictures compare to the theory?

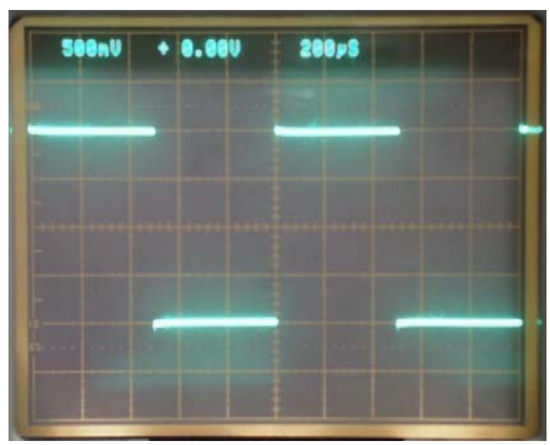

Figure 2-11: Oscillograph of 1 kHz Square Wave
(Vertical, 500 mV/Div; Horizontal, 200 σS/Div)

The oscilloscope picture is exactly as we expected. There's no apparent DC level, and the period is 5 divisions, which works out to be 1000 σs or 1 ms. The frequency of this waveform is therefore:

$$f = \frac{1}{T} = \frac{1}{1\,ms} = \underline{\underline{1\,kHz}}$$

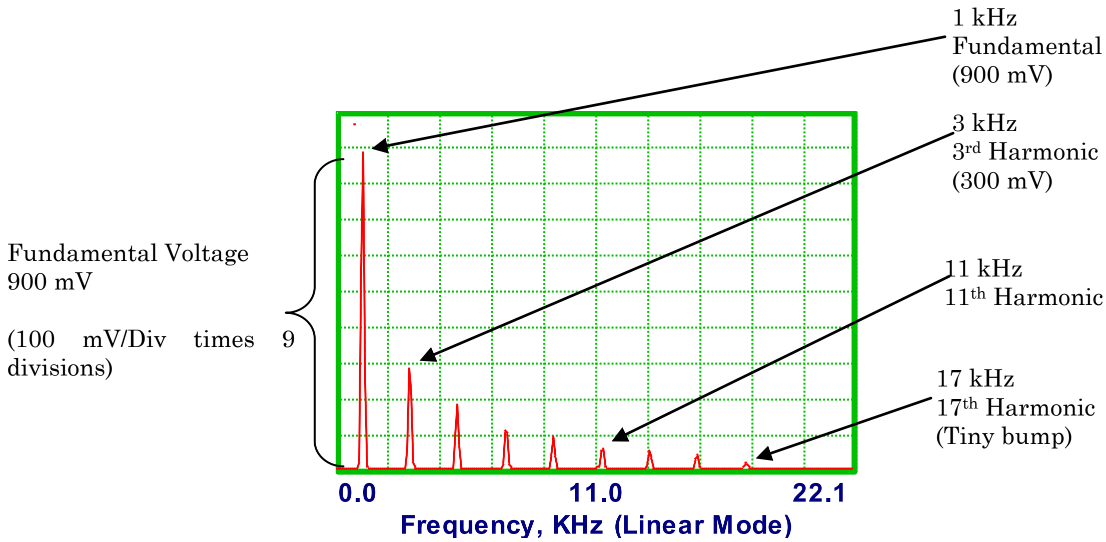

Figure 2-12: Spectrogram of a Square Wave (Vertical Setting, 100 mV/Division)

The spectrum analyzer view also agrees with the theory. The fundamental frequency is 1 kHz and measures 900 mV. The third harmonic measures only 300 mV, the fifth (5 kHz) measures 200 mV, and so on. Notice how the signals start getting small around 11 kHz. *This is why most technicians stop analysis at the 13th harmonic*. The 17th harmonic is very small, and barely visible as a bump on the graph.

Separating Square Wave Signal Components

There's another way of understanding the square wave of Figures 2-7 and 2-11. A square wave is nothing more than the sum of an infinite number of sine waves, each with a frequency that is an odd multiple of the fundamental frequency. If we were to graph these sine waves and add them together point by point, we would get the square wave. Figure 2-13 shows how this works.

One way of thinking about the Fourier analysis of a waveform is to consider it a recipe for building that particular signal in the *frequency domain*. To obtain the *resultant* wave of Figure 2-13, all the points in the *fundamental, third harmonic, and fifth harmonic* are added together.

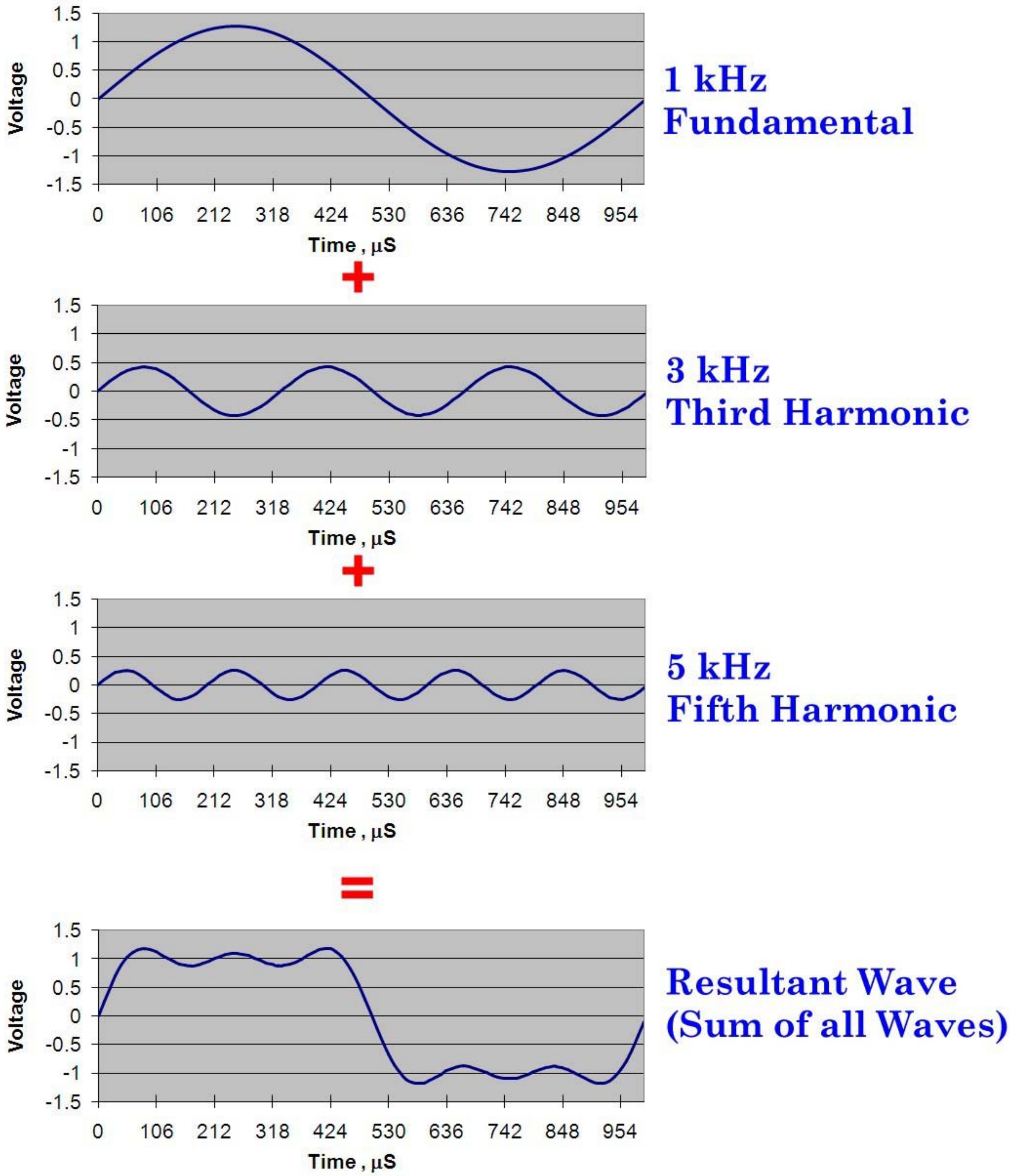

Figure 2-13: The Recipe for a Square Wave in the Time Domain